Exercise 5.1

1. In which of the following situations, does the list of numbers involved make as arithmetic progression and why?

(i) The taxi fare after each km when the fare is Rs 15 for the first km and Rs 8 for each additional km.

Sol. Taxi fare for 1 km = 15

Taxi fare for first 2 kms = 15 + 8 = 23

Taxi fare for first 3 kms = 23 + 8 = 31

Taxi fare for first 4 kms = 31 + 8 = 39

And so on……

∴The sequence is 15, 23, 31, 39 … forms an A.P.

Because every next term is 8 more than the preceding term.

(ii) The amount of air present in a cylinder when a vacuum pump removes 1/4 of the air remaining in the cylinder at a time.

Sol. Let the volume of air in the cylinder is a unit.

Vacuum pump removes = $\frac{a}{4}$ unit

∴ Remaining air in cylinder = a - $\frac{a}{4}$ = $\frac{3a}{4}$

Again, the Vacuum pump removes = $\frac{1}{4}$ of $\frac{3a}{4}$ unit

= $\frac{3a}{16}$ unit

∴ Remaining air in cylinder = $\frac{3a}{4}$ - $\frac{3a}{16}$

= $\frac{12a-3a}{16}$

= $\frac{9a}{16}$ unit

Therefore, volumes will be a, $\frac{3a}{4}$, $\frac{9a}{16}$, … and so on

Here the common difference is not same.

∴ This series is not an A.P. Ans.

(iii) The cost of digging a well after every metre of digging, when it costs Rs 150 for the first metre and rises by Rs 50 for each subsequent metre.

Sol. Cost of digging the well for first metre = Rs.150

Cost of digging the well for first 2 metres = Rs.150 + 50 = Rs.200

Cost of digging the well for first 3 metres = Rs.200 + 50 = Rs.250

Cost of digging the well for first 4 metres =Rs.250 + 50 = Rs.300

And so on...

∴ The sequence is 150, 200, 250, 300, … forms an A.P.
Because every next term is 50 more than the preceding term.

(iv) The amount of money in the account every year, when Rs 10000 is deposited at compound interest at 8% per annum.

Sol. Principal = Rs. 10000

Rate of interest (r) = 8% per annum

Amount after 1 year = $P \times \left(1+\frac{r}{100}\right)^t$

$= 10000 \times \left(1+\frac{8}{100}\right)^1$

$= 10000 \times \frac{108}{100}$

$= 10800$

Amount after 2 years = $10000 \times \left(1+\frac{8}{100}\right)^2$

$= 10000 \times \frac{108}{100} \times \frac{108}{100}$

$= 11664$

Amount after 3 years = $10000 \times \left(1+\frac{8}{100}\right)^3$

$= 10000 \times \frac{108}{100} \times \frac{108}{100} \times \frac{108}{100}$

$= 12597.12$

Therefore, the amount of money will be;

10800, 11664, 12597.12, ……. and so, on

Here the common difference is not same.

∴ This series is not an A.P. Ans.

2. Write first four terms of the A.P. when the first term a and the common difference d are given as follows:

(i) a = 10, d = 10

Sol. Here a = 10 and d = 10

$\therefore a_2 = a + d = 10 + 10 = 20$

$a_3 = a + 2d = 10 + 20 = 30$

$a_4 = a + 3d = 10 + 30 = 40$ and so on…

∴ First four terms of this A.P. are 10, 20, 30, and 40. Ans.

(ii) a = -2, d = 0
Sol. Here a = -2 and d = 0
$\therefore a_2 = a + d = -2 + 0 = -2$
$a_3 = a + 2d = -2 + 2 \times 0 = -2$
$a_4 = a + 3d = -2 + 3 \times 0 = -2$ and so on...
∴ First four terms of this A.P. are -2, -2, -2, and -2. Ans.

(iii) a = 4, d = – 3
Sol. Here a = 4 and d = -3
$\therefore a_2 = a + d = 4 + (-3) = 1$
$a_3 = a + 2d = 4 + 2 \times (-3) = 4 - 6 = -2$
$a_4 = a + 3d = 4 + 3 \times (-3) = 4 - 9 = -5$ and so on...
∴ First four terms of this A.P. are 4, 1, -2, and -5 Ans.

(iv) $a = -1\ d = \frac{1}{2}$
Sol. Here a = -1 and $d = \frac{1}{2}$
$\therefore a_2 = a + d = -1 + \frac{1}{2} = \frac{-1}{2}$
$a_3 = a + 2d = -1 + 2 \times \frac{1}{2} = -1 + 1 = 0$
$a_4 = a + 3d = -1 + 3 \times \frac{1}{2} = -1 + \frac{3}{2} = \frac{1}{2}$ and so on...
∴ First four terms of this A.P. are $-1, \frac{-1}{2}, 0$, and $\frac{1}{2}$ Ans.

(v) a = – 1.25, d = – 0.25
Sol. Here a = – 1.25 and d = – 0.25
$\therefore a_2 = a + d = -1.25 + (-0.25) = -1.50$
$a_3 = a + 2d = -1.25 + 2 \times (-0.25) = -1.25 - 0.50 = -1.75$
$a_4 = a + 3d = -1.25 + 3 \times (-0.25) = -1.25 - 0.75 = -2.00$ and so on...
∴ First four terms of this A.P. are – 1.25, -1.50, -1.75, and -2.00 Ans.

3. For the following A.P.s, write the first term and the common difference.
(i) 3, 1, – 1, – 3 ...
Sol. The given A.P. is 3, 1, -1, -3, ...
Here a = 3
and $d = a_2 - a_1 = 1 - 3 = -2$
∴ a = 3 and d = -2 Ans.

(ii) -5, – 1, 3, 7 …

Sol. The given A.P. is -5, – 1, 3, 7 …

Here a = -5

and d = $a_2 - a_1$

= -1 – (-5)

= -1 + 5 = 4

∴ a = - 5 and d = 4 Ans.

(iii) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}$….

Sol. The given A.P. is $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}$….

Here a = $\frac{1}{3}$

and d = $a_2 - a_1$

= $\frac{5}{3} - \frac{1}{3}$

= $\frac{4}{3}$

∴ a = $\frac{1}{3}$ and d = $\frac{4}{3}$ Ans.

(iv) 0.6, 1.7, 2.8, 3.9 …

Sol. The given A.P. is 0.6, 1.7, 2.8, 3.9 …

Here a = 0.6

and d = $a_2 - a_1$

= 1.7 – 0.6

= 1.1

∴ a = 0.6 and d = 1.1 Ans.

4. Which of the following are APs? If they form an A.P. find the common difference d and write three more terms.

(i) 2, 4, 8, 16 …

Sol. The given series is 2, 4, 8, 16 …

Here, $a_2 - a_1$ = 4 – 2 = 2

and $a_3 - a_2$ = 8 – 4 = 4

Here $a_2 - a_1 \neq a_3 - a_2$

∴ The given series is not forming an A. P. Ans.

(ii) 2, $\frac{5}{2}$, 3, $\frac{7}{2}$ …

Sol. The given series is 2, $\frac{5}{2}$, 3, $\frac{7}{2}$ …

Here, $a_2 - a_1 = \frac{5}{2} - 2 = \frac{1}{2}$

and $a_3 - a_2 = 3 - \frac{5}{2} = \frac{1}{2}$

Here $a_2 - a_1 = a_3 - a_2$

∴ The given series is forming an A. P.

For next three terms

$a_5 = a + 4d = 2 + 4 \times \frac{1}{2} = 2 + 2 = 4$

$a_6 = a + 5d = 2 + 5 \times \frac{1}{2} = 2 + \frac{5}{2} = \frac{9}{2}$

$a_7 = a + 6d = 2 + 6 \times \frac{1}{2} = 2 + 3 = 5$

∴ Next three terms of this A.P. are 4, $\frac{9}{2}$ and 5 Ans.

(iii) -1.2, -3.2, -5.2, -7.2 ...

Sol. The given series is -1.2, -3.2, -5.2, -7.2 ...

Here, $a_2 - a_1$ = -3.2 - (-1.2) = -3.2 + 1.2 = -2

and $a_3 - a_2$ = -5.2 - (-3.2) = -5.2 + 3.2 = -2

Here $a_2 - a_1 = a_3 - a_2$

∴ The given series is forming an A. P.

For next three terms

$a_5 = a + 4d$ = - 1.2 + 4 × (-2) = -1.2 - 8 = - 9.2

$a_6 = a + 5d$ = - 1.2+ 5 × (-2) = -1.2 - 10.0 = - 11.2

$a_7 = a + 6d$ = - 1.2 + 6 × (-2) = -1.2 - 12 = - 13.2

∴ Next three terms of this A.P. are - 9.2, - 11.2 and - 13.2 Ans.

(iv) - 10, - 6, - 2, 2 ...

Sol. The given series is -10, - 6, - 2, 2 ...

Here, $a_2 - a_1$ = - 6 - (- 10) = - 6 + 10 = 4

and $a_3 - a_2$ = - 2 - (- 6) = - 2 + 6 = 4

Here $a_2 - a_1 = a_3 - a_2$

∴ The given series is forming an A. P.

For next three terms

$a_5 = a + 4d$ = -10 + 4 × 4 = -10 + 16 = 6

$a_6 = a + 5d$ =-10 + 5 × 4 = -10 + 20 = 10

$a_7 = a + 6d$ = -10 + 6 × 4 = -10 + 24 = 14

∴ Next three terms of this A.P. are 6, 10 and 14 Ans.

(v) $3, 3 + \sqrt{2}, 3 + 2\sqrt{2}, 3 + 3\sqrt{2}$

Sol. The given series is $3, 3 + \sqrt{2}, 3 + 2\sqrt{2}, 3 + 3\sqrt{2}$...

Here, $a_2 - a_1 = 3 + \sqrt{2} - 3 = \sqrt{2}$

and $a_3 - a_2 = 3 + 2\sqrt{2} - (3 + \sqrt{2}) = 3 + 2\sqrt{2} - 3 - \sqrt{2} = \sqrt{2}$

Here $a_2 - a_1 = a_3 - a_2$

∴ The given series is forming an A. P.

For next three terms

$a_5 = a + 4d = 3 + 4 \times \sqrt{2} = 3 + 4\sqrt{2}$

$a_6 = a + 5d = 3 + 5 \times \sqrt{2} = 3 + 5\sqrt{2}$

$a_7 = a + 6d = 3 + 6 \times \sqrt{2} = 3 + 6\sqrt{2}$

∴ Next three terms of this A.P. are $3 + 4\sqrt{2}$, $3 + 5\sqrt{2}$ and $3 + 6\sqrt{2}$ Ans.

(vi) 0.2, 0.22, 0.222, 0.2222 ....

Sol. The given series is 0.2, 0.22, 0.222, 0.2222 ....

Here, $a_2 - a_1 = 0.22 - 0.2 = 0.02$

and $a_3 - a_2 = 0.222 - 0.22 = 0.002$

Here $a_2 - a_1 \neq a_3 - a_2$

∴ The given series is not forming an A. P. Ans.

(vii) 0, – 4, – 8, – 12 ...

Sol. The given series is 0, – 4, – 8, – 12 ...

Here, $a_2 - a_1 = -4 - 0 = -4$

$a_3 - a_2 = -8 - (-4) = -8 + 4 = -4$

Here $a_2 - a_1 = a_3 - a_2$

∴ The given series is forming an A. P.

For next three terms

$a_5 = a + 4d = 0 + 4 \times (-4) = -16$

$a_6 = a + 5d = 0 + 5 \times (-4) = -20$

$a_7 = a + 6d = 0 + 6 \times (-4) = -24$

∴ Next three terms of this A.P. are - 16, -20 and -24 Ans.

(viii) $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}$....

Sol. The given series is $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}$....

Here, $a_2 - a_1 = -\frac{1}{2} - (-\frac{1}{2}) = -\frac{1}{2} + \frac{1}{2} = 0$

and $a_3 - a_2 = -\frac{1}{2} - (-\frac{1}{2}) = -\frac{1}{2} + \frac{1}{2} = 0$

Here $a_2 - a_1 = a_3 - a_2$

∴ The given series is forming an A. P.

For next three terms

$a_5 = a + 4d = -\frac{1}{2} + 4 \times 0 = -\frac{1}{2} + 0 = -\frac{1}{2}$

$a_6 = a + 5d = -\frac{1}{2} + 5 \times 0 = -\frac{1}{2} + 0 = -\frac{1}{2}$

$a_7 = a + 6d = -\frac{1}{2} + 6 \times 0 = -\frac{1}{2} + 0 = -\frac{1}{2}$

∴ Next three terms of this A.P. are $-\frac{1}{2}$, $-\frac{1}{2}$ and $-\frac{1}{2}$ Ans.

(ix) 1, 3, 9, 27 ...

Sol. The given series is 1, 3, 9, 27 ...

Here, $a_2 - a_1 = 3 - 1 = 2$

and $a_3 - a_2 = 9 - 3 = 6$

Here $a_2 - a_1 \neq a_3 - a_2$

∴ The given series is not forming an A. P. Ans.

(x) a, 2a, 3a, 4a ...

Sol. The given series is a, 2a, 3a, 4a ...

Here, $a_2 - a_1 = 2a - a = a$

and $a_3 - a_2 = 3a - 2a = a$

Here $a_2 - a_1 = a_3 - a_2$

∴ The given series is forming an A. P.

For next three terms

$a_5 = a + 4d = a + 4 \times a = a + 4a = 5a$

$a_6 = a + 5d = a + 5 \times a = a + 5a = 6a$

$a_7 = a + 6d = a + 6 \times a = a + 6a = 7a$

∴ Next three terms of this A.P. are 5a, 6a and 7a Ans.

(xi) a, a^2, a^3, a^4 ...

Sol. The given series is a, a^2, a^3, a^4 ...

Here, $a_2 - a_1 = a^2 - a = a(a - 1)$

and $a_3 - a_2 = a^3 - a^2 = a^2(a - 1)$

Here $a_2 - a_1 \neq a_3 - a_2$

∴ The given series is not forming an A. P. Ans.

(xii) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}$ ...

Sol. The given series is $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}$ ...

Or, $\sqrt{2}, 2\sqrt{2}, 3\sqrt{2}, 4\sqrt{2}$ ...

Here, $a_2 - a_1 = 2\sqrt{2} - \sqrt{2} = \sqrt{2}$

and $a_3 - a_2 = 3\sqrt{2} - 2\sqrt{2} = \sqrt{2}$

Here $a_2 - a_1 = a_3 - a_2$

∴ The given series is forming an A. P.

For next three terms

$a_5 = a + 4d = \sqrt{2} + 4 \times \sqrt{2} = \sqrt{2} + 4\sqrt{2} = 5\sqrt{2} = \sqrt{50}$

$a_6 = a + 5d = \sqrt{2} + 5 \times \sqrt{2} = \sqrt{2} + 5\sqrt{2} = 6\sqrt{2} = \sqrt{72}$

$a_7 = a + 6d = \sqrt{2} + 6 \times \sqrt{2} = \sqrt{2} + 6\sqrt{2} = 7\sqrt{2} = \sqrt{98}$

∴ Next three terms of this A.P. are $\sqrt{50}, \sqrt{72}$ and $\sqrt{98}$ Ans.

(xiii) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}$ ...

Sol. The given series is $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}$ ...

Here, $a_2 - a_1 = \sqrt{6} - \sqrt{3} = \sqrt{3} \times \sqrt{2} - \sqrt{3} = \sqrt{3}(\sqrt{2} - 1)$

and $a_3 - a_2 = \sqrt{9} - \sqrt{6} = 3 - \sqrt{6} = \sqrt{3}(\sqrt{3} - \sqrt{2})$

Here $a_2 - a_1 \neq a_3 - a_2$

∴ The given series is not forming an A. P. Ans.

(xiv) $1^2, 3^2, 5^2, 7^2$ ...

Sol. The given series is $1^2, 3^2, 5^2, 7^2$ ...

Or, 1, 9, 25, 49, ...

Here, $a_2 - a_1 = 9 - 1 = 8$

and $a_3 - a_2 = 25 - 9 = 16$

Here $a_2 - a_1 \neq a_3 - a_2$

∴ The given series is not forming an A. P. Ans.

(xv) $1^2, 5^2, 7^2, 73$ ...

Sol. The given series is $1^2, 5^2, 7^2, 73$ ...

Or 1, 25, 49, 73 ...

Here, $a_2 - a_1 = 25 - 1 = 24$

and $a_3 - a_2 = 49 - 25 = 24$

Here $a_2 - a_1 = a_3 - a_2$

∴ The given series is forming an A. P.

For next three terms

$a_5 = a + 4d = 1 + 4 \times 24 = 1 + 96 = 97$

$a_6 = a + 5d = 1 + 5 \times 24 = 1 + 120 = 121$
$a_7 = a + 6d = 1 + 6 \times 24 = 1 + 144 = 145$
$\therefore$ Next three terms of this A.P. are 97, 121 and 145 Ans.

Exercise 5.2

1. Fill in the blanks in the following table, given that a is the first term, d the common difference and a_n is the nth term of the A.P.

a	d	n	a_n
7	3	8	---
-18	---	10	0
---	-3	18	-5
-18.9	2.5	---	3.6
3.5	0	105	---

Sol. (i) Here, $a = 7$, $d = 3$ and $n = 8$,
$a_n = a + (n - 1) d$
$\therefore a_8 = 7 + (8 - 1) 3$
$= 7 + 7 \times 3$
$= 7 + 21$
$= 28$
hence, $a_8 = 28$ Ans.

(ii) Here, $a = -18$, $d = ?$ and $n = 10$
$a_n = 0$
$\Rightarrow a + (n - 1) d = 0$
$\Rightarrow -18 + (10 - 1) d = 0$
$\Rightarrow 9 d = 18$
$\Rightarrow d = 2$
Hence, $d = 2$ Ans.

(iii) Here $a = ?$ $d = -3$, $n = 18$
and $a_n = -5$
$\Rightarrow a + (n - 1) d = -5$
$\Rightarrow a + (18 - 1) (-3) = -5$
$\Rightarrow a - 54 + 3 = -5$
$\Rightarrow a - 51 = -5$

$\Rightarrow a = -5 + 51$
$\Rightarrow a = 46$
Hence, a = 46 Ans.
(iv) Here, a = -18.9, d = 2.5, n =?
and $a_n = 3.6$
$\Rightarrow a + (n - 1) d = 3.6$
$\Rightarrow -18.9 + (n - 1) 2.5 = 3.6$
$\Rightarrow -18.9 + 2.5 n - 2.5 = 3.6$
$\Rightarrow -21.4 + 2.5 n = 3.6$
$\Rightarrow 2.5 n = 3.6 + 21.4$
$\Rightarrow 2.5 n = 25.0$
$\Rightarrow n = 10$
Hence, n = 10

(v) Here, a = 3.5, d = 0, n = 105
$a_n = a + (n - 1) d$
$a_{105} = 3.5 + (105 - 1)0$
$= 3.5 + 104 \times 0$
$= 3.5$
Hence, $a_{105} = 3.5$ Ans.

2. Choose the correct choice in the following and justify:
(i) 30^{th} term of the A.P: 10,7, 4, …, is
(a) 97 (b) 77 (c) –77 (d) –87
Sol. The given A.P. is 10, 7, 4, …
Here, a = 10 and $d = a_2 - a_1$
$= 7 - 10 = -3$
$a_n = a + (n - 1) d$
$a_{30} = 10 + (30 - 1) (-3)$
$= 10 + (29) (-3)$
$= 10 - 87$
$= -77$
Hence, the correct answer is option c.

(ii) 11^{th} term of the A.P. $-3, -\frac{1}{2}, 2$ …. is
(a) 28 (b) 22 (c) – 38 (d)
Sol. The given A.P. is $-3, -\frac{1}{2}, 2$ ….

Here, a = -3 and d = $a_2 - a_1$

$= -\frac{1}{2} + 3 = \frac{5}{2}$

$a_n = a + (n - 1)\, d$

$a_{11} = -3 + (11 - 1)\frac{5}{2}$

$= -3 + 10 \times \frac{5}{2}$

$= -3 + 25$

$= 22$

Hence, the correct answer is option b.

3. In the following APs find the missing term in the boxes.

(i) 2, □, 26

Sol. Here a = 2

And $a_3 = 26$

$\Rightarrow a + (n - 1)\, d = 26$

$\Rightarrow 2 + (3 - 1)\, d = 26$

$\Rightarrow 2d = 26 - 2$

$\Rightarrow 2d = 24$

$\Rightarrow d = 12$

$a_2 = 2 + (2 - 1)12$

$= 14$

$\therefore$ The missing term is 14. Ans.

(ii) □, 13, □, 3

Sol. Here $a_2 = 13$ and $a_4 = 3$

$\Rightarrow a + d = 13$ ..... (i)

And $a + 3d = 3$ ..... (ii)

On subtracting equation (i) from (ii), we get

$2d = -10$

$\Rightarrow d = -5$

Put the value of 'd' in equation (i), we have,

$a + (-5) = 13$

$\Rightarrow a = 13 + 5$

$\Rightarrow a = 18$

Now, $a_3 = a + 2d$

$= 18 + 2 \times (-5)$

$= 18 - 10 = 8$

∴ The missing term are 18 and 8. Ans.

(iii) 5,□,□, $9\frac{1}{2}$

Sol. Here a = 5

and $a_4 = 9\frac{1}{2}$

$\Rightarrow a + 3d = \frac{19}{2}$ .....(i)

$\Rightarrow 5 + 3d = \frac{19}{2}$

$\Rightarrow 3d = \frac{19}{2} - 5$

$\Rightarrow 3d = \frac{9}{2}$

$\Rightarrow d = \frac{3}{2}$

Put the value of 'd' in equation (i), we have,

$a + 3 \times \frac{3}{2} = \frac{19}{2}$

$\Rightarrow a + \frac{9}{2} = \frac{19}{2}$

$\Rightarrow a = \frac{19}{2} - \frac{9}{2}$

$\Rightarrow a = \frac{10}{2}$

$\Rightarrow a = 5$

Now, $a_2 = a + d$

$= 5 + \frac{3}{2}$

$= \frac{13}{2}$

and $a_3 = a + 2d$

$= 5 + 2 \times \frac{3}{2}$

$= 5 + 3$

$= 8$

∴ The missing term are $\frac{13}{2}$ and 8. Ans.

(iv) -4,□,□,□,□,6

Sol. Here a = -4

and $a_6 = 6$

$\Rightarrow a + 5d = 6$ .....(i)

$\Rightarrow -4 + 5d = 6$

$\Rightarrow 5d = 6 + 4$

$\Rightarrow 5d = 10$

$\Rightarrow d = 2$

Now, $a_2 = a + d$

$= -4 + 2$

$= -2$

$a_3 = a + 2d$

$= -4 + 2 \times 2$

$= -4 + 4$

$= 0$

$a_4 = a + 3d$

$= -4 + 3 \times 2$

$= -4 + 6$

$= 2$

$a_5 = a + 4d$

$= -4 + 4 \times 2$

$= -4 + 8$

$= 4$

∴ The missing term are -2, 0, 2 and 4. Ans.

(v) □,38,□,□,□,-22

Sol. Here $a_2 = 38$ and $a_6 = -22$

$\Rightarrow a + d = 38$ ..... (i)

And $a + 5d = -22$ ..... (ii)

On subtracting equation (i) from (ii), we get

$4d = -60$

$\Rightarrow d = -15$

Put the value of 'd' in equation (i), we have,

$a + (-15) = 38$

$\Rightarrow a = 38 + 15$

$\Rightarrow a = 53$

Now, $a_3 = a + 2d$

$= 53 + 2 \times (-15)$

$= 53 - 30$

$= 23$

$a_4 = a + 3d$

$= 53 + 3 \times (-15)$

$= 53 - 45$

$= 8$

$a_5 = a + 4d$

$= 53 + 4 \times (-15)$

$= 53 - 60$

= - 7

∴ The missing term are 53, 23, 8 and -7 respectively. Ans.

4. Which term of the A.P. 3, 8, 13, 18, … is 78?

Sol. The given A.P. is 3, 8, 13, 18, …

Here, $a = 3$, $d = a_2 - a_1$

$= 8 - 3 = 5$

and $a_n = 78$

$\Rightarrow a + (n - 1) d = 78$

$\Rightarrow 3 + (n - 1) 5 = 78$

$\Rightarrow 3 + 5n - 5 = 78$

$\Rightarrow 5n - 2 = 78$

$\Rightarrow 5n = 78 + 2$

$\Rightarrow 5n = 80$

$\Rightarrow n = 16$

Hence, 16th term of this A.P. is 78. Ans.

5. Find the number of terms in each of the following A.P.

(i) 7, 13, 19, …, 205

Sol. The given A.P. is 7, 13, 19, …, 205

Here, $a = 7$, $d = a_2 - a_1$

$= 13 - 7 = 6$

and $a_n = 205$

$\Rightarrow a + (n - 1) d = 205$

$\Rightarrow 7 + (n - 1) 6 = 205$

$\Rightarrow 7 + 6n - 6 = 205$

$\Rightarrow 6n + 1 = 205$

$\Rightarrow 6n = 205 - 1$

$\Rightarrow 6n = 204$

$\Rightarrow n = 34$

Hence, 34th term of this A.P. is 205. Ans.

(ii) 18, $15\frac{1}{2}$, 13, …, -47

Sol. The given A. P. is 18, $15\frac{1}{2}$, 13, …, -47

Here $a = 18$, $d = a_2 - a_1 = 15\frac{1}{2} - 18 = -2\frac{1}{2}$

And $a_n = -47$

$\Rightarrow a + (n - 1) d = - 47$

$\Rightarrow 18 + (n - 1)\left(\frac{-5}{2}\right) = -47$

$\Rightarrow 18 - \frac{5}{2}n + \frac{5}{2} = -47$

$\Rightarrow -\frac{5}{2}n = -47 - 18 - \frac{5}{2}$

$\Rightarrow -\frac{5}{2}n = \frac{-94 - 36 - 5}{2}$

$\Rightarrow -\frac{5}{2}n = \frac{-135}{2}$

$\Rightarrow n = \frac{135}{2} \times \frac{2}{5}$

$\Rightarrow n = 27$

Hence, 27th term of this A.P. is -47. Ans.

6. Check whether -150 is a term of the A.P. 11, 8, 5, 2, ...

Sol. The given series is 11, 8, 5, 2,

Here $a = 11$, $d = a_2 - a_1 = 8 - 11 = -3$

And $a_n = -150$

$\Rightarrow a + (n - 1)d = -150$

$\Rightarrow 11 + (n - 1)(-3) = -150$

$\Rightarrow 11 - 3n + 3 = -150$

$\Rightarrow 14 - 3n = -150$

$\Rightarrow -3n = -150 - 14$

$\Rightarrow -3n = -164$

$\Rightarrow n = \frac{164}{3}$

$\Rightarrow n = 54.8$

Clearly, n is not a natural number.

$\therefore$ - 150 is not a term of this A.P. Ans.

7. Find the 31st term of an A.P. whose 11th term is 38 and the 16th term is 73.

Sol. Here $a_{11} = 38$ and $a_{16} = 73$

$\Rightarrow a + 10d = 38$ ..... (i)

And $a + 15d = 73$ ..... (ii)

On subtracting equation (i) from (ii), we get

$5d = 35$

$\Rightarrow d = 7$

Put the value of 'd' in equation (i), we have,

$a + 10 \times 7 = 38$

$\Rightarrow a + 70 = 38$

$\Rightarrow a = 38 - 70$

$\Rightarrow a = -32$

Now, $a_{31} = a + (31-1)\,d$

$= -32 + 30 \times 7$

$= -32 + 210$

$= 178$

$\therefore$ 31st term is 178. Ans.

8. An A.P. consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29th term.

Sol. Here $a_3 = 12$, $a_n = 106$ and $n = 50$

$\Rightarrow a + 2d = 12$ ..... (i)

And $a + (n-1)\,d = 106$

Or, $a + (50-1)\,d = 106$

Or, $a + 49\,d = 106$ ..... (ii)

On subtracting equation (i) from (ii), we get

$49d - 2d = 106 - 12$

$\Rightarrow 47d = 94$

$\Rightarrow d = 2$

Put the value of 'd' in equation (i), we have,

$a + 2 \times 2 = 12$

$\Rightarrow a + 4 = 12$

$\Rightarrow a = 12 - 4$

$\Rightarrow a = 8$

Now, $a_{29} = a + (29-1)\,d$

$= 8 + 28 \times 2$

$= 8 + 56$

$= 64$

$\therefore$ 29th term is 64. Ans.

9. If the 3rd and the 9th terms of an A.P. are 4 and – 8 respectively. Which term of this A.P. is zero.

Sol. Here $a_3 = 4$ and $a_9 = -8$

$\Rightarrow a + 2d = 4$ ..... (i)

And $a + 8d = -8$ ..... (ii)

On subtracting equation (i) from (ii), we get

$8d - 2d = -8 - 4$

$\Rightarrow 6d = -12$

$\Rightarrow d = -2$

Put the value of 'd' in equation (i), we have,

$a + 2 \times (-2) = 4$
$\Rightarrow a - 4 = 4$
$\Rightarrow a = 4 + 4$
$\Rightarrow a = 8$
Now, $a_n = 0$
$\Rightarrow a + (n - 1)\, d = 0$
$\Rightarrow 8 + (n - 1)\,(-2) = 0$
$\Rightarrow 8 - 2n + 2 = 0$
$\Rightarrow 10 - 2n = 0$
$\Rightarrow - 2n = - 10$
$\Rightarrow n = 5$
$\therefore 5^{th}$ term of the A. P. is 0. Ans.

10. If 17th term of an A.P. exceeds its 10th term by 7. Find the common difference.
Sol. Here $a_{17} = a_{10} + 7$
$\Rightarrow a + 16d = a + 9d + 7$
$\Rightarrow a - a + 16d - 9d = 7$
$\Rightarrow 7d = 7$
$\Rightarrow d = 1$
$\therefore$ The common difference is 1. Ans.

11. Which term of the A.P. 3, 15, 27, 39, ... will be 132 more than its 54th term?
Sol. The given A.P. is 3, 15, 27, 39, ……
Here $a = 7$ and $d = a_2 - a_1$
$= 15 - 3 = 12$
A. t. q. $a_n = a_{54} + 132$
$\Rightarrow a + (n - 1)\, d = a + (54 - 1)\, d + 132$
$\Rightarrow a + nd - d - a - 53\, d = 132$
$\Rightarrow nd - 54d = 132$
$\Rightarrow n \times 12 - 54 \times 12 = 132$
$\Rightarrow 12\, n - 648 = 132$
$\Rightarrow 12\, n = 132 + 648$
$\Rightarrow 12\, n = 780$
$\Rightarrow n = 65$

∴ 65th term is 132 more than its 54th term. Ans.

12. Two APs have the same common difference. The difference between their 100th term is 100, what is the difference between their 1000th terms?

Sol. Let the A. P. are a, a + d, a + 2d, ……
And A, A + d, A + 2d, ……
A. t. q. The difference between their 100th term is 100
$\Rightarrow a_n - A_n = 100$
$\Rightarrow [a + (n - 1) d] - [A + (n - 1) d] = 100$
$\Rightarrow [a + 99d] - [A + 99d] = 100$
$\Rightarrow a + 99d - A - 99d = 100$
$\Rightarrow a - A = 100$ ……(i)
Now, difference between 1000th terms of the two APs
$= a_{1000} - A_{1000}$
$= [a + (1000 - 1) d] - [A + (1000 - 1) d]$
$= [a + 999d] - [A + 999d]$
$= a + 99d - A - 99d$
= a – A from (i)
= 100
∴ The difference between 1000th terms of the two A.P. is 100. Ans.

13. How many three-digit numbers are divisible by 7?

Sol. Three-digit number which are divisible by 7 are
105, 112, 119, 126, ……, 994
Here a = 105 and $d = a_2 - a_1$
$= 112 - 105$
$= 7$
Now, $a_n = 994$
$\Rightarrow a + (n - 1) d = 994$
$\Rightarrow 105 + (n - 1) 7 = 994$
$\Rightarrow 105 + 7n - 7 = 994$
$\Rightarrow 98 + 7n = 994$
$\Rightarrow 7n = 994 - 98$
$\Rightarrow 7n = 896$
$\Rightarrow n = 128$
∴ There are 128 three-digit numbers which are divisible by 7. Ans.

14. How many multiples of 4 lie between 10 and 250?

Sol. The multiples of 4 and lie between 10 and 250 are

12, 16, 20, ……, 248

Here $a = 12$ and $d = a_2 - a_1$

$= 16 - 12 = 4$

Now, $a_n = 248$

$\Rightarrow a + (n - 1) d = 248$

$\Rightarrow 12 + (n - 1) 4 = 248$

$\Rightarrow 12 + 4n - 4 = 248$

$\Rightarrow 8 + 4n = 248$

$\Rightarrow 4n = 248 - 8$

$\Rightarrow 4n = 240$

$\Rightarrow n = 60$

$\therefore$ There are 60 multiples of 4 between 10 and 250. Ans.

15. For what value of n, are the nth terms of two APs 63, 65, 67, and 3, 10, 17, … equal?

Sol. Two APs 63, 65, 67, and 3, 10, 17, …

In first A.P., $a = 63$, $d = 65 - 63 = 2$

$a_n = a + (n - 1) d$

$= 63 + (n - 1) 2$

$= 63 + 2n - 2$

$= 61 + 2n$

In second A.P., $A = 3$, $D = 10 - 3 = 7$

$A_n = A + (n - 1) D$

$= 3 + (n - 1) 7$

$= 3 + 7n - 7$

$= 7n - 4$

A. t. q. nth term of both A.P. are equal

$\Rightarrow 61 + 2n = 7n - 4$

$\Rightarrow 2n - 7n = - 61 - 4$

$\Rightarrow - 5n = - 65$

$\Rightarrow n = 13$

$\therefore$ 13th terms of both these A.P.s are equal to each other. Ans.

16. Determine the A.P. whose third term is 16 and the 7th term exceeds the 5th term by 12.

Sol. Here $a_3 = 16$

$\Rightarrow a + 2d = 16$ ..... (i)

And $a_7 = a_5 + 12$

$\Rightarrow a + 6d = a + 4d + 12$

$\Rightarrow a + 6d - a - 4d = 12$

$\Rightarrow 2d = 12$

$\Rightarrow d = 6$..... (ii)

Put the value of 'd' in equation (i), we have,

$a + 2 \times 6 = 16$

$\Rightarrow a + 12 = 16$

$\Rightarrow a = 16 - 12$

$\Rightarrow a = 4$

Now, $a_2 = a + d$

$= 4 + 6$

$= 10$

$\therefore$ A.P. will be 4, 10, 16, 22, ... Ans.

17. Find the 20th term from the last term of the A.P. 3, 8, 13, ..., 253.

Sol. The given A.P. is 3, 8, 13, ..., 253

20th term from the last term of the A.P. 3, 8, 13, ..., 253

means 20th term from the beginning of the A.P. 253, 248, 243, ..., 13, 8, 5

$\therefore$ A.P. is 253, 248, 243, ..., 13, 8, 5

Here $a = 253$, $n = 20$ and $d = 248 - 253 = -5$

$a_n = a + (n - 1) d$

$a_{20} = 253 + (20 - 1)(-5)$

$= 253 + 19 \times (-5)$

$= 253 - 95$

$= 158$

$\therefore$ The 20th term from the last term of the AP 3, 8, 13, ..., 253 is 158. Ans.

18. The sum of 4th and 8th terms of an A.P. is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms of the A.P.

Sol. Here $a_4 + a_8 = 24$

$\Rightarrow a + 3d + a + 7d = 24$

$\Rightarrow 2a + 10d = 24$

$\Rightarrow a + 5d = 12$ ..... (i)

And $a_6 + a_{10} = 44$

$\Rightarrow a + 5d + a + 9d = 44$

$\Rightarrow 2a + 14d = 44$

$\Rightarrow a + 7d = 22$ ..... (ii)

On subtracting equation (i) from (ii), we get

$7d - 5d = 22 - 12$

$\Rightarrow 2d = 10$

$\Rightarrow d = 5$

Put the value of 'd' in equation (i), we have,

$a + 5 \times 5 = 12$

$\Rightarrow a + 25 = 12$

$\Rightarrow a = 12 - 25$

$\Rightarrow a = -13$

Now, $a_2 = a + d = -13 + 5 = -8$

and $a_3 = a_2 + d = -8 + 5 = -3$

$\therefore$ The first three terms of this A.P. are –13, –8, and –3.

19. Subba Rao started work in 1995 at an annual salary of Rs 5000 and received an increment of Rs 200 each year. In which year did his income reach Rs 7000?

Sol. It can be seen from the given question, that the incomes of Subba Rao increases every year by Rs.200 and hence, forms an AP.

Salary in 1st year = Rs. 5000

Salary in 2nd year = Rs. 5000 + Rs. 200 = Rs. 5200

Salary in 3rd year = Rs. 5200 + Rs. 200 = Rs. 5400 and so on...

This forms an A.P. 5000, 5200, 5400, ...

With $a = 5000$ and $d = 200$

Here $a_n = 7000$

$\Rightarrow a + (n - 1) d = 7000$

$\Rightarrow 5000 + (n - 1) 200 = 7000$

$\Rightarrow 5000 + 200 n - 200 = 7000$

$\Rightarrow 4800 + 200 n = 7000$

$\Rightarrow 200 n = 7000 - 4800$

$\Rightarrow 200 n = 2200$

$\Rightarrow n = 11$

∴ In 11th year, his salary will be Rs 7000. Ans.

20. Ramkali saved Rs 5 in the first week of a year and then increased her weekly saving by Rs 1.75. If in the nth week, her weekly savings become Rs 20.75, find n.

Sol. Ramkali saved in 1^{st} week = Rs.5

in 2^{nd} week = Rs. 5 + Rs. 1.75 = Rs. 6.75

in 3^{rd} week = Rs. 6.75 + Rs. 1.75 = Rs. 8.50 and so on .....

This forms an A.P. 5, 6.75, 8.50, ...

With a = 5 and d = 1.75

Here $a_n = 20.75$

$\Rightarrow a + (n - 1)\, d = 20.75$

$\Rightarrow 5 + (n - 1)\, 1.75 = 20.75$

$\Rightarrow 5 + 1.75\, n - 1.75 = 20.75$

$\Rightarrow 1.75\, n + 3.25 = 20.75$

$\Rightarrow 1.75\, n = 20.75 - 3.25$

$\Rightarrow 1.75\, n = 17.50$

$\Rightarrow n = 10$

∴ In 10^{th} week, her saving will be Rs 20.75. Ans.

Exercise 5.3

1. Find the sum of the following APs.

(i) 2, 7, 12, ..., to 10 terms.

Sol. The given A.P. is 2, 7, 12, ...

Here, a = 2, $d = a_2 - a_1$

$= 7 - 2 = 5$

$\therefore S_n = \frac{n}{2}[2a + (n - 1)\, d]$

$\Rightarrow S_{10} = \frac{10}{2}[2 \times 2 + (10 - 1) \times 5]$

$= 5[4 + 9 \times 5]$

$= 5[4 + 45]$

$= 5 \times 49$

$= 245$

∴ Sum of 10 terms is 245. Ans.

(ii) – 37, – 33, – 29, ... to 12 terms

Sol. The given A.P. is – 37, – 33, – 29, ...

Here, a = -37, $d = a_2 - a_1$

$= -33 - (-37)$

$= -33 + 37 = 4$

$\therefore S_n = \frac{n}{2}[2a + (n - 1)d]$

$\Rightarrow S_{12} = \frac{12}{2}[2 \times (-37) + (12 - 1) \times 4]$

$= 6[-74 + 11 \times 4]$

$= 6[-74 + 44]$

$= 6 \times (-30)$

$= -180$

$\therefore$ Sum of 12 terms is -180. Ans.

(iii) 0.6, 1.7, 2.8, ... to 100 terms

Sol. The given A.P. is 0.6, 1.7, 2.8, ...

Here, $a = 0.6$, $d = a_2 - a_1$

$= 1.7 - 0.6 = 1.1$

$\therefore S_n = \frac{n}{2}[2a + (n - 1)d]$

$\Rightarrow S_{100} = \frac{100}{2}[2 \times 0.6 + (100 - 1) \times 1.1]$

$= 50[1.2 + 99 \times 1.1]$

$= 50[1.2 + 108.9]$

$= 50 \times 110.1$

$= 5505$

$\therefore$ Sum of 100 terms is 5505. Ans.

(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \ldots\ldots$ to 11 terms

Sol. The given A.P. is $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \ldots\ldots$

Here, $a = \frac{1}{15}$, $d = a_2 - a_1$

$= \frac{1}{12} - \frac{1}{15} = \frac{1}{60}$

$\therefore S_n = \frac{n}{2}[2a + (n - 1)d]$

$\Rightarrow S_{11} = \frac{11}{2}[2 \times \frac{1}{15} + (11 - 1) \times \frac{1}{60}]$

$= \frac{11}{2}[\frac{2}{15} + \frac{1}{6}]$

$= \frac{11}{2}[\frac{4 + 5}{30}]$

$= \frac{11}{2} \times \frac{9}{30}$

$= \frac{11}{2} \times \frac{3}{10}$

$= \frac{33}{20}$

$\therefore$ Sum of 100 terms is $\frac{33}{20}$. Ans.

2. Find the sums given below:

(i) $7 + 10\frac{1}{2} + 14 + \ldots\ldots\ldots\ldots + 84$

Sol. Here the given series is

Here, $a = 7$, $d = \frac{7}{2}$

and $a_n = 84$

$\Rightarrow a + (n - 1)\, d = 84$

$\Rightarrow 7 + (n - 1)\frac{7}{2} = 84$

$\Rightarrow 7 + \frac{7}{2}n - \frac{7}{2} = 84$

$\Rightarrow \frac{7}{2} n = 84 - 7 + \frac{7}{2}$

$\Rightarrow \frac{7}{2} n = \frac{168 - 14 + 7}{2}$

$\Rightarrow \frac{7}{2} n = \frac{161}{2}$

$\Rightarrow n = \frac{161}{2} \times \frac{2}{7}$

$\Rightarrow n = 23$

$\therefore S_n = \frac{n}{2}[a + l]$

$\Rightarrow S_{23} = \frac{23}{2}[7 + 84]$

$= \frac{23}{2} \times 91$

$= \frac{2093}{2}$

$\therefore$ Sum of the series is $\frac{2093}{2}$ Ans.

(ii) $34 + 32 + 30 + \ldots\ldots\ldots + 10$

Sol. Here the given series is $34 + 32 + 30 + \ldots\ldots\ldots + 10$

Here, $a = 34$, $d = 32 - 34 = -2$

and $a_n = 10$

$\Rightarrow a + (n - 1)\, d = 10$

$\Rightarrow 34 + (n - 1)(-2) = 10$

$\Rightarrow 34 - 2n + 2 = 10$

$\Rightarrow 36 - 2n = 10$

$\Rightarrow -2n = 10 - 36$

$\Rightarrow -2n = -26$

$\Rightarrow n = 13$

$\therefore S_n = \frac{n}{2}[a + l]$

$\Rightarrow S_{13} = \frac{13}{2}[34 + 10]$

$= \frac{13}{2} \times 44$

$= 13 \times 22$

$= 286$

$\therefore$ Sum of the series is 286. Ans.

(iii) $-5 + (-8) + (-11) + \ldots\ldots\ldots + (-230)$

Sol. Here the given series is $-5 + (-8) + (-11) + \ldots\ldots\ldots + (-230)$

Here, $a = -5$, $d = -8 - (-5) = -8 + 5 = -3$

and $a_n = -230$

$\Rightarrow a + (n - 1)d = -230$

$\Rightarrow -5 + (n - 1)(-3) = -230$

$\Rightarrow -5 - 3n + 3 = -230$

$\Rightarrow -2 - 3n = -230$

$\Rightarrow -3n = -230 + 2$

$\Rightarrow -3n = -228$

$\Rightarrow n = 76$

$\therefore S_n = \frac{n}{2}[a + l]$

$\Rightarrow S_{76} = \frac{76}{2}[-5 + (-230)]$

$= 38 \times (-235)$

$= -8930$

$\therefore$ Sum of the series is -8930. Ans.

3. In an AP

(i) Given $a = 5$, $d = 3$, $a_n = 50$, find n and Sn.

Sol. Here, $a = 5$, $d = 3$

and $a_n = 50$

$\Rightarrow a + (n - 1)d = 50$

$\Rightarrow 5 + (n - 1)3 = 50$

$\Rightarrow 5 + 3n - 3 = 50$

$\Rightarrow 2 + 3n = 50$

$\Rightarrow 3n = 50 - 2$

$\Rightarrow 3n = 48$
$\Rightarrow n = 16$
$\therefore S_n = \frac{n}{2}[a + a_n]$

$\Rightarrow S_{16} = \frac{16}{2}[5 + 50]$

$= 8 \times 55$
$= 440$
$\therefore$ n is 16 and S_{16} is 440. Ans.

(ii) Given $a = 7$, $a_{13} = 35$, find d and S_{13}.
Sol. Here, $a = 7$
and $a_{13} = 35$
$\Rightarrow a + (n - 1)\, d = 35$
$\Rightarrow 7 + (13 - 1)\, d = 35$
$\Rightarrow 12d = 35 - 7$
$\Rightarrow 12d = 28$
$\Rightarrow d = \frac{7}{3}$
$\therefore S_n = \frac{n}{2}[2a + (n - 1)\, d]$

$\Rightarrow S_{13} = \frac{13}{2}[2 \times 7 + (13 - 1)\frac{7}{3}]$

$= \frac{13}{2}[14 + 12 \times \frac{7}{3}]$
$= \frac{13}{2}[14 + 28]$
$= \frac{13}{2} \times 42$
$= 13 \times 21$
$= 273$
$\therefore$ d is $\frac{7}{3}$ and S_{13} is 273. Ans.

(iii) Given $a_{12} = 37$, $d = 3$, find a and S_{12}.
Sol. Here, $d = 3$
And $a_{12} = 37$
$\Rightarrow a + (12 - 1)\, 3 = 37$
$\Rightarrow a + 11 \times 3 = 37$
$\Rightarrow a + 33 = 37$
$\Rightarrow a = 37 - 33$

$\Rightarrow a = 4$

$\therefore S_n = \frac{n}{2}[a + a_n]$

$\Rightarrow S_{12} = \frac{12}{2}[4 + 37]$

$= 6 \times 41$

$= 246$

$\therefore$ a is 4 and S_{12} is 246. Ans.

(iv) Given $a_3 = 15$, $S_{10} = 125$, find d and a_{10}.

Sol. Here, $a_3 = 15$

$\Rightarrow a + 2d = 15$ .....(i)

and $S_{10} = 125$

$\Rightarrow \frac{10}{2}[2a + (10 - 1)d] = 125$

$\Rightarrow 5[2a + 9d] = 125$

$\Rightarrow 2a + 9d = 25$.....(ii)

Multiply (i) by 2, $2a + 4d = 30$

Multiply (ii) by 1, $2a + 9d = 25$

On subtracting $-5d = 5$

$\Rightarrow d = -1$

Put the value of 'd' in equation (i)

$\Rightarrow a + 2(-1) = 15$

$\Rightarrow a - 2 = 15$

$\Rightarrow a = 15 + 2$

$\Rightarrow a = 17$

Now, $a_n = a + (n - 1)d$

$\therefore a_{10} = 17 + (10 - 1)(-1)$

$= 17 + 9 \times (-1)$

$= 17 - 9$

$= 8$

$\therefore$ d is -1 and a_{10} is 8. Ans.

(v) Given $d = 5$, $S_9 = 75$, find a and a_9.

Sol. Here, $d = 5$

and $S_9 = 75$

$\Rightarrow \frac{9}{2}[2a + (9 - 1)5] = 75$ [$\because S_n = \frac{n}{2}\{2a + (n - 1)d\}$]

$\Rightarrow \frac{9}{2}[2a + 40] = 75$

$\Rightarrow 9a + 180 = 75$

$\Rightarrow 9a = 75 - 180$

$\Rightarrow 9a = -105$

$\Rightarrow a = -\frac{105}{9}$

$\Rightarrow a = -\frac{35}{3}$

Now, $a_n = a + (n - 1)\, d$

$\therefore a_9 = -\frac{35}{3} + (9 - 1)\, 5$

$= -\frac{35}{3} + 8 \times 5$

$= -\frac{35}{3} + 40$

$= \frac{-35 + 120}{3}$

$= \frac{85}{3}$

$\therefore a$ is $-\frac{35}{3}$ and a_9 is $\frac{85}{3}$. Ans.

(vi) Given $a = 2$, $d = 8$, $S_n = 90$, find n and a_n

Sol. Here, $a = 2$, $d = 8$

and $S_n = 90$

$\Rightarrow \frac{n}{2}\{2a + (n - 1)\, d\} = 90$

$\Rightarrow \frac{n}{2}\{2 \times 2 + (n - 1)\, 8\} = 90$

$\Rightarrow \frac{n}{2}\{4 + 8n - 8\} = 90$

$\Rightarrow \frac{n}{2}\{8n - 4\} = 90$

$\Rightarrow 4n^2 - 2n - 90 = 0$

$\Rightarrow 2n^2 - n - 45 = 0$

$\Rightarrow 2n^2 - 10n + 9n - 45 = 0$

$\Rightarrow 2n(n - 5) + 9(n - 5) = 0$

$\Rightarrow (n - 5)(2n + 9) = 0$

$\Rightarrow n - 5 = 0$ or $2n + 9 = 0$

$\Rightarrow n = 5$ or $n = \frac{-9}{2}$

$\therefore n = 5$ ($\because n$ never be negative or fraction)

Now, $a_n = a + (n - 1)\, d$

$\therefore a_5 = 2 + (5 - 1)\, 8$

$= 2 + 4 \times 8$

$= 2 + 40$

$= 42$

$\therefore$ n is 5 and a_5 is 42. Ans.

(vii) Given, a = 8, a_n = 62, S_n = 210, find n and d

Sol. Here, a = 8

And a_n = 8

$\Rightarrow$ a + (n – 1) d = 62

$\Rightarrow$ 8 + (n – 1) d = 62

$\Rightarrow$ (n – 1) d = 62 - 8

$\Rightarrow$ (n – 1) d = 54 ..... (i)

and S_n = 210

$\Rightarrow \frac{n}{2}$ {2a + (n – 1) d} = 210

$\Rightarrow \frac{n}{2}$ {2 × 8 + 54} = 210 from (i)

$\Rightarrow \frac{n}{2}$ {16 + 54} = 210

$\Rightarrow \frac{n}{2}$ × 70 = 210

$\Rightarrow$ 35 n = 210

$\Rightarrow$ n = 6

Put the value of 'n' in equation (i)

(6 – 1) d = 54

$\Rightarrow$ 5d = 54

$\Rightarrow$ d = $\frac{54}{5}$

$\therefore$ n is 6 and d is 10.8. Ans.

(viii) given a_n = 4, d = 2, S_n = –14, find n and a.

Sol. Here, a_n = 4, d = 2 and S_n = –14

Now, a_n = 4

$\Rightarrow$ a + (n – 1) d = 4

$\Rightarrow$ a + (n – 1) 2 = 4

$\Rightarrow$ a + 2n – 2 = 4

$\Rightarrow$ a + 2n = 4 + 2

$\Rightarrow$ a = 6 - 2n..... (i)

and S_n = –14

$\Rightarrow \frac{n}{2}$ {a + a_n} = -14

$\Rightarrow \frac{n}{2}$ {6 - 2n + 4} = -14

$\Rightarrow \frac{n}{2}$ {10 – 2n} = -14

$\Rightarrow 5n - n^2 = -14$

$\Rightarrow n^2 - 5n - 14 = 0$

$\Rightarrow n^2 - 7n + 2n - 14 = 0$

$\Rightarrow n(n - 7) + 2(n - 7) = 0$

$\Rightarrow (n - 7)(n + 2) = 0$

$\Rightarrow n - 7 = 0$ or $n + 2 = 0$

$\Rightarrow n = 7$ or $n = -2$

$\therefore n = 7$ ($\because$ n never be negative)

Put the value of 'n' in equation (i)

$a = 6 - 2n$

$= 6 - 2 \times 7$

$= 6 - 14$

$= -8$

$\therefore$ n is 7 and d is -8. Ans.

(ix) given $a = 3$, $n = 8$, $S_n = 192$, find d

Sol. Here $a = 3$, $n = 8$,

And, $S_n = 192$

$\Rightarrow \frac{n}{2}[2a + (n - 1)d] = 192$

$\Rightarrow \frac{8}{2}[2 \times 3 + (8 - 1)d] = 192$

$\Rightarrow 4[6 + 7d] = 192$

$\Rightarrow 24 + 28d = 192$

$\Rightarrow 28d = 192 - 24$

$\Rightarrow 28d = 168$

$\Rightarrow d = 6$

$\therefore$ d is 6. Ans.

(x) given $l = 28$, $S = 144$, and there are total 9 terms. Find a

Sol. Here, $l = a_n = 28$, and $S_n = 144$

Now, $a_9 = 28$ [$\because$ total terms are 9]

$\Rightarrow a + (9 - 1)d = 28$

$\Rightarrow a + 8d = 28$

$\Rightarrow a = 28 - 8d$.....(i)

and $S_9 = 144$

$\Rightarrow \frac{9}{2}[a + 28] = 144$ [$\because S_n = \frac{n}{2}\{a + a_n\}$]

$\Rightarrow a + 28 = 144 \times \frac{2}{9}$

$\Rightarrow a + 28 = 16 \times 2$

$\Rightarrow a = 32 - 28$

$\Rightarrow a = 4$

$\therefore$ a is 4. Ans.

4. How many terms of the AP. 9, 17, 25 ... must be taken to give a sum of 636?

Sol. The given A. P. is 9, 17, 25 ...

Here, $a = 9$, $d = 17 - 9 = 8$

and $S_n = 636$

$\Rightarrow \frac{n}{2}[2a + (n - 1)\, d] = 636$

$\Rightarrow \frac{n}{2}[2 \times 9 + (n - 1)\, 8] = 636$

$\Rightarrow \frac{n}{2}[18 + 8n - 8] = 636$

$\Rightarrow \frac{n}{2}[10 + 8n] = 636$

$\Rightarrow 5n + 4n^2 = 636$

$\Rightarrow 4n^2 + 5n - 636 = 0$

$\Rightarrow 4n^2 + 53n - 48n - 636 = 0$

$\Rightarrow n(4n + 53) - 12(4n + 53) = 0$

$\Rightarrow (4n + 53)(n - 12) = 0$

$\Rightarrow 4n + 53 = 0$ or $n - 12 = 0$

$\Rightarrow n = \frac{-53}{4}$ or $n = 12$

$\therefore n = 5$ ($\because$ n never be negative or fraction)

$\therefore$ Sum of 12 terms is 636. Ans.

5. The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

Sol. Here, $a = 5$, $l = a_n = 45$,

and $S_n = 400$

$\Rightarrow \frac{n}{2}[5 + 45] = 400 \qquad [\because S_n = \frac{n}{2}\{a + a_n\}]$

$\Rightarrow \frac{n}{2} \times 50 = 400$

$\Rightarrow 25\, n = 400$

$\Rightarrow n = 16$

And $a_n = 45$

$\Rightarrow a + (n - 1)\, d = 45$

$\Rightarrow 5 + (16 - 1)\, d = 45$
$\Rightarrow 15\, d = 45 - 5$
$\Rightarrow 15\, d = 40$
$\Rightarrow d = \frac{40}{15}$
$\Rightarrow d = \frac{8}{3}$
$\therefore$ The number of terms is 16 and the common difference is $\frac{8}{3}$ Ans.

6. The first and the last term of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?
Sol. Here, $a = 17$, $l = a_n = 350$ and $d = 9$
Now, $a_n = 350$
$\Rightarrow a + (n - 1)\, d = 350$
$\Rightarrow 17 + (n - 1)\, 9 = 350$
$\Rightarrow 17 + 9n - 9 = 350$
$\Rightarrow 8 + 9n = 350$
$\Rightarrow 9\, n = 350 - 8$
$\Rightarrow 9\, n = 342$
$\Rightarrow n = 38$
and $S_n = \frac{n}{2}[a + a_n]$
$\Rightarrow S_{38} = \frac{38}{2}[17 + 350]$
$= 19 \times 367$
$= 6973$
$\therefore$ The number of terms is 38 and sum of the terms is 6973. $\frac{8}{3}$ Ans.

7. Find the sum of first 22 terms of an AP in which d = 7 and 22nd term is 149.
Sol. Here, $n = 22$, $d = 7$,
and $a_{22} = 149$
$\Rightarrow a + (22 - 1)\, 7 = 149$ [$\because a_n = a + (n - 1)\, d$]
$\Rightarrow a + 21 \times 7 = 149$
$\Rightarrow a + 147 = 149$
$\Rightarrow a = 149 - 147$
$\Rightarrow a = 2$
$\therefore S_n = \frac{n}{2}[2a + (n - 1)\, d]$

$\Rightarrow S_{13} = \frac{22}{2}[2 \times 2 + (22 - 1)\, 7]$

$= 11 \times [4 + 21 \times 7]$
$= 11 \times [4 + 147]$
$= 11 \times 151$
$= 1661$

$\therefore$ a is 2 and S_{13} is 1661. Ans.

8. Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.

Sol. Here n = 51, $a_2 = 14$

$\Rightarrow a + d = 14$ .....(i)

And $a_3 = 18$

$\Rightarrow a + 2d = 18$ .....(ii)

Subtract (ii) from (i), we have

$d = 4$

Put the value of 'd' in equation (i), we have

$a + 4 = 14$

$\Rightarrow a = 14 - 4$

$\Rightarrow a = 10$

$\therefore S_n = \frac{n}{2}[2a + (n - 1)\, d]$

$\Rightarrow S_{51} = \frac{51}{2}[2 \times 10 + (51 - 1)\, 4]$

$= \frac{51}{2} \times [20 + 50 \times 4]$
$= \frac{51}{2} \times [20 + 200]$
$= \frac{51}{2} \times 220$
$= 51 \times 110$
$= 5610$

$\therefore$ Sum of 51 terms is 5610. Ans.

9. If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

Sol. Here sum of first 7 terms of an AP is 49

$\Rightarrow S_7 = 49$

$\Rightarrow \frac{7}{2}[2a + (7 - 1)\, d] = 49$

$\Rightarrow 2a + 6d = 49 \times \frac{2}{7}$

$\Rightarrow 2a + 6d = 14$

$\Rightarrow a + 3d = 7$ ..... (i)

And $S_{17} = 289$

$\Rightarrow \frac{17}{2}[2a + (17 - 1)d] = 289$

$\Rightarrow 2a + 16d = 289 \times \frac{2}{17}$

$\Rightarrow 2a + 16d = 34$

$\Rightarrow a + 8d = 17$ ..... (ii)

Subtract (i) from (ii), we have

$5d = 10$

$\Rightarrow d = 2$

Put the value of 'd' in equation (i), we have

$a + 3 \times 2 = 7$

$\Rightarrow a + 6 = 7$

$\Rightarrow a = 7 - 6$

$\Rightarrow a = 1$

$\therefore S_n = \frac{n}{2}[2a + (n - 1)d]$

$= \frac{n}{2}[2 \times 1 + (n - 1)2]$

$= \frac{n}{2}[2 + 2n - 2]$

$= \frac{n}{2} \times 2n$

$= n^2$

$\therefore$ The sum of first n terms is n^2. Ans.

10. Show that $a_1, a_2 \ldots, a_n$, form an AP where a_n is defined as below
(i) $a_n = 3+4n$ (ii) $a_n = 9-5n$
Also find the sum of the first 15 terms in each case.

Sol. (i) $a_n = 3 + 4n$

For $n = 1, a_1 = 3 + 4 \times 1 = 7$

$n = 2, a_2 = 3 + 4 \times 2 = 3 + 8 = 11$

$n = 3, a_3 = 3 + 4 \times 3 = 3 + 12 = 15$

$n = 4, a_4 = 3 + 4 \times 4 = 3 + 16 = 19$

$\therefore$ The sequence is 7, 11, 15, 19, ...

$a_2 - a_1 = 11 - 7 = 4$

$a_3 - a_2 = 15 - 11 = 4$

Here $a_2 - a_1 = a_3 - a_2$

$\therefore$ This sequence is forming an A. P.

Now, $S_n = \frac{n}{2}[2a + (n - 1)d]$

$\therefore S_{15} = \frac{15}{2}[2 \times 7 + (15 - 1)4]$
$= \frac{15}{2}[14 + 14 \times 4]$
$= \frac{15}{2}[14 + 56]$
$= \frac{15}{2} \times 70$
$= 15 \times 35$
$= 525$

$\therefore$ The sum of first 15 terms is 525. Ans.

Sol. (ii) $a_n = 9 - 5n$
For $n = 1, a_1 = 9 - 5 \times 1 = 9 - 5 = 4$
$n = 2, a_2 = 9 - 5 \times 2 = 9 - 10 = -1$
$n = 3, a_3 = 9 - 5 \times 3 = 9 - 15 = -6$
$n = 4, a_4 = 9 - 5 \times 4 = 9 - 20 = -11$
$\therefore$ The sequence is 4, -1, -6, -11, ...
$a_2 - a_1 = -1 - 4 = -5$
$a_3 - a_2 = -6 - (-1) = -6 + 1 = -5$
Here $a_2 - a_1 = a_3 - a_2$
$\therefore$ This sequence is forming an A. P.
Now, $S_n = \frac{n}{2}[2a + (n - 1)d]$

$\therefore S_{15} = \frac{15}{2}[2 \times 4 + (15 - 1)(-5)]$
$= \frac{15}{2}[8 + 14 \times (-5)]$
$= \frac{15}{2}[8 - 70]$
$= \frac{15}{2} \times (-62)$
$= 15 \times (-31)$
$= -465$

$\therefore$ The sum of first 15 terms is -465. Ans.

11. If the sum of the first n terms of an AP is $4n - n^2$, what is the first term (that is S_1)? What is the sum of first two terms? What is the second term? Similarly find the 3rd, the 10th and the nth terms.
Sol. Here, $S_n = 4n - n^2$
For $n = 1, S_1 = 4 \times 1 - 1^2$
$= 4 - 1 = 3$

$\Rightarrow a = 3$ [$\because a_1 = S_1$]

$n = 2, S_2 = 4 \times 2 - 2^2$
$= 8 - 4 = 4$
$\Rightarrow a_2 = 4 - 3$ [$\because a_2 = S_2 - S_1$]
$= 1$
$n = 3, S_3 = 4 \times 3 - 3^2$
$= 12 - 9 = 3$
$\Rightarrow a_3 = 3 - 4$ [$\because a_3 = S_3 - S_2$]
$= -1$

$\therefore$ The A.P. is 3, 1, -1, ...
Here $a = 3$ and $d = 1 - 3 = -2$
$a_n = a + (n - 1) d$
$\therefore a_{10} = 3 + (10 - 1)(-2)$
$= 3 + 9 \times (-2)$
$= 3 - 18$
$= -15$
And $a_n = 3 + (n - 1)(-2)$
$= 3 - 2n + 2$
$= 5 - 2n$

$\therefore$ First term is 3, second term is 1, third term is -1, sum of two terms is 4, tenth term is -15 and the sum of n terms is $5 - 2n$. Ans.

12. Find the sum of first 40 positive integers divisible by 6.
Sol. The positive integers which are divisible by 6 are
6, 12, 18, 24 ....
Here, $a = 6$, $d = 6$ and $n = 40$
$\therefore S_n = \frac{n}{2}[2a + (n - 1) d]$

$S_{40} = \frac{40}{2}[2 \times 6 + (40 - 1) 6]$
$= 20 [12 + 39 \times 6]$
$= 20 [12 + 234]$
$= 20 \times 246$
$= 4920$

$\therefore$ The sum of first 40 positive integers divisible by 6 is 4920. Ans.

13. Find the sum of first 15 multiples of 8.
Sol. First 15 multiples of 8 are

8, 16, 24, 32……, 120

Here, a = 8, d = 8 and n = 15

$\therefore S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{15} = \frac{15}{2}[2 \times 8 + (15-1)8]$

$= \frac{15}{2}[16 + 14 \times 8]$

$= \frac{15}{2}[16 + 112]$

$= \frac{15}{2} \times 128$

$= 15 \times 64$

$= 960$

∴ The sum of first 15 multiples of 8 is 960. Ans.

14. Find the sum of the odd numbers between 0 and 50.

Sol. The odd numbers between 0 and 50 are

1, 3, 5, 7, 9 … 49.

Here, a = 1, d = 2

and $a_n = 49$

$\Rightarrow a + (n-1)d = 49$

$\Rightarrow 1 + (n-1)2 = 49$

$\Rightarrow 1 + 2n - 2 = 49$

$\Rightarrow 2n - 1 = 49$

$\Rightarrow 2n = 49 + 1$

$\Rightarrow 2n = 50$

$\Rightarrow n = 25$

Now, $S_n = \frac{n}{2}[a + l]$

$\therefore S_{25} = \frac{25}{2}[1 + 49]$

$= \frac{25}{2} \times 50$

$= 25 \times 25$

$= 625$

∴ The sum of the odd numbers between 0 and 50 is 625. Ans.